Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at http://about.jstor.org/participate-jstor/individuals/early-journal-content.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

Substituting the value of R we have $\pi r^2(\pi--2\theta)+4\pi\theta r^2\cos^2\theta-\pi r^2\sin 2\theta=\frac{1}{2}\pi^2r^2$ or $2\theta\cos 2\theta-\sin 2\theta=\frac{1}{2}\pi$, from which, $\theta=54^\circ\ 35'\ 39''$,

$\therefore\quad R=2r\cos\theta=8.269265$ rods.

Solutions were also received from *J. F. W. Scheffer, H. C. Whitaker, G. B. M. Zerr, and I. L. Beverage.*

CALCULUS.

Conducted by **J. M. COLAW, Monterey, Va.** All contributions to this department should be sent to him.

SOLUTIONS TO PROBLEMS.

22. Proposed by **MOSES C. STEVENS, M. A., Professor of Mathematics, Purdue University, Lafayette, Indiana.**

Solve the following Differential Equation:

$$(6x^3+20x^2-2x)\frac{d^2y}{dx^2}-(9x^2+10x+1)\frac{dy}{dx}+(1+9x)y=0.$$

I. **Solution by E. B. ESCOTT, Ann Arbor, Michigan.**

This can be written $(6x^3+20x^2-2x)\dfrac{d^2y}{dx^2}-(1+9x)(1+x)\dfrac{dy}{dx}+(1+9x)y=0$

It is evident from inspection that $y=1+x$ is a particular solution.

Assume, therefore, $y=v\,(1+x)$.

Substituting in the given equation, we have

$$\frac{d^2v}{dx^2}+\left(\frac{2}{1+x}-\frac{9x^2+10x+1}{6x^3+20x^2-2x}\right)\frac{dv}{dx}=0.$$

Let $\dfrac{dv}{dx}=z$, then $\dfrac{d^2v}{dx^2}=\dfrac{dz}{dx}$. $\dfrac{dz}{dx}+\left(\dfrac{2}{1+x}-\dfrac{9x^2+10x+1}{6x^3+20x^2-2x}\right)z=0.$

$$\frac{dz}{z}+\left(\frac{2}{1+x}-\frac{9x^2+10x+1}{6x^3+20x^2-2x}\right)dx=0.$$

Integrating, $z=\dfrac{c(3x^2+10x-1)}{(1+x)^2\sqrt{x}}$ or $dv=\dfrac{c(3x^2+10x-1)}{(1+x)^2\sqrt{x}}\,dx.$

Integrating, $v=c\sqrt{x}\left(\dfrac{3x-1}{x+1}\right)+c'$. $\therefore\quad y=c(3x-1)\sqrt{x}+c'(1+x).$

II. **Solution by G. B. M. ZERR, A. M., Principal of High School, Staunton, Virginia.**

The equation may be written as follows:

$$\frac{d^2y}{dx^2}-\frac{9x^2+10x+1}{6x^3+20x^2-2x}\frac{dy}{dx}+\frac{9x+1}{6x^3+20x^2-2x}y=0,$$

Let $y=v(x+1)$; then the transformed equation is

$$\frac{d^2v}{dx^2}+\left\{\frac{2}{1+x}-\frac{9x^2+10x+1}{6x^3+20x^2-2x}\right\}\frac{dv}{dx}=0$$

$$\therefore\ \frac{dv}{dx}(1+x)^2e^{-\int\frac{9x^2+10x+1}{6x^3+20x^2-2x}dx}=A.$$

$$\therefore\ \frac{dv}{dx}=\frac{A}{(1+x)^2}e^{\int\frac{9x^2+10x+1}{6x^3+20x^2-2x}dx}=\frac{3x^2+10x-1}{x^{\frac{1}{2}}(1+x)^2}A.$$

$$\therefore\quad V=\frac{6x^{\frac{3}{2}}-2x^{\frac{1}{2}}}{1+x}A+B.\qquad \text{But } y=v(x+1),\quad \because\ y=2x^{\frac{1}{2}}(3x-1)A+(1+x)B,$$

a complete solution, where A and B are arbitrary constants.

Particular solutions are $y=1+x$ and $y=2x^{\frac{1}{2}}(3x-1)$.

III. **Solution by WILLIAM HOOVER, A. M., Ph. D., Professor of Mathematics and Astronomy, Ohio University, Athens, Ohio, and the PROPOSER.**

Assume $y=A_0x^m\ldots.(1)$; that the given equation becomes

$(6m^2-15m+9)x^{m+1}+(20m^2-30m+1)x^m-(2m^2-m)x^{m-1}=0\ldots.(2)$.

If $m'=m-1\ldots.(3)$, $m'+5=m\ldots.(4)$, $\kappa=1\ldots.(5)$.

Put, then, $y=\Sigma_0^\infty A_rx^{m+r}\ldots.(6)$; and the given equation becomes again

$$\Sigma_0^\infty\big\{[6(m+r)(m+r-2)-9(m+r)+9]A_rx^{m+r+1}$$
$$+[20(m+r)(m+r-1)-10(m+r)+1]A_rx^{m+r}$$
$$-[2(m+r)(m+r-1)+(m+r)]A_rx^{m+r-1}\big\}=0\ldots.(7).$$

Putting for r, $r-2$, $r-1$ in the first two terms, (7) becomes

$$\Sigma_0^\infty\big\{[6(m+r-1)(m+r-3)-9(m+r-2)+9]A_{r-2}$$
$$+[20(m+r-1)(m+r-2)-10(m+r-1)+1]A_{r-1}$$
$$-[2(m+r)(m+r-1)+(m+r)]A_r\big\}x^{m+r-1}=0\ldots.(8).$$

Making the coefficient of A_r vanish after putting $r=0$, we have $m=0\ldots.(9)$, $m=\frac{1}{2}\ldots.(10)$.

Substituting $m=0$ in the first two terms, and writing $r+2$ for r, $(6r^2-15r+9)A_r+(20r^2+10r-9)A_{r+1}=0\ldots.(11)$. Putting $r=0$, $r=1$ in succession in (11), $A_1=A_0$, $A_2=0$; then one part of the integral is $y'=A_0(1+x)\ldots.(12)$. The like step for $m=\frac{1}{2}$ gives another part of the integral $y''=B_0x^{\frac{1}{2}}(1-3x)\ldots.(13)$.

The whole integral is $y=y'+y''=A_0(1+x)+B_0x^{\frac{1}{2}}(1-3x)\ldots.(14)$.

[This result is also correct since the constant $A'=my-2A$. This has proved a very interesting problem. Prof. Matz sent four different solutions.—Ed.]

Also solved by *H. C. WHITAKER* and *C. E. WHITE*.

23. Proposed by W. I. TAYLOR, Baldwin University, Berea, Ohio.

From a point O situated in the plane of a plane curve, radii vectores are drawn to different points of the curve, and on each one a distance is laid off from O inversely proportional to the length of the radius vector: to determine the tangent at any point of the locus of the points thus obtained. [*Byerly's Diff. Calculus*, p. 177, ex. 1.]